UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4240 – Game theory and economics of information**

Date of exam: Tuesday, May 26, 2009 **Grades will be given: ##**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers three pages

Note: You can give your answer in English or Norwegian!

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of *five* problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1 (10 %)

True or false? For each of the statements, if true, try to explain why, and if false, provide a counterexample.

(a) If a finite normal-form game has only one rationalizable strategy for each player, then there cannot be more than one Nash equilibrium.

(b) In a finite normal-form game with a unique Nash equilibrium, each player has a unique rationalizable strategy.

(c) In a finite extensive-form game of perfect information, there always exists a unique subgame perfect Nash equilibrium.

(d) In any game of perfect information, every Nash equilibrium is also a subgame perfect equilibrium.

Problem 2 (20 %)

Consider a two-player game in which the strategy spaces are $S_1 = S_2 = [0, \infty)$. That is, each player selects a number that is greater than or equal to zero. Let s_1 denote the strategy of player 1 and let s_2 denote the strategy of player 2. Suppose that the payoff functions are given by

$$u_1(s_1, s_2) = 2s_1 + 2as_1s_2 - s_1^2$$

and

$$u_2(s_1, s_2) = 2s_2 + 2as_1s_2 - s_2^2$$

where a is a constant parameter.

(a) Is there any value of a such that this game has no Nash equilibrium? If so, provide such a value. In either case, explain your answer.

(b) Is there any value of a such that this game has an efficient Nash equilibrium? If so, provide such a value. In either case, explain your answer.

Problem 3 (10 %)

In the following version of the "Battle of the Sexes", show that there is a unique mixed-strategy Nash equilibrium and compute it. (Player 1 chooses row and player 2 chooses column. Denote by p the probability that player 1 chooses O and by q the probability that player 2 chooses O.) Do you think that the mixed-strategy equilibrium is a more appealing outcome for this game compared to the two pure-strategy equilibria? Why or why not?

	O	F
O	5,2	0,0
F	0,0	2,5

Problem 4 (20 %)

Consider the following infinitely-repeated game: Each period a principal and an agent contract for the agent to produce a good of quality q, which costs the agent $c(q)$, where $c'(q) > 0$ and $c''(q) > 0$. If the good of the agreed quality q is delivered, the principal is to pay t to the agent. The principal's payoff from the transaction is $q - t$, and the agent's is $t - c(q)$. Both the principal and agent are risk neutral. The value of each player's outside option is zero. In each period, the players agree on q and t, then the principal must write a contract, and then they play the production/trade game. The principal incurs a cost k of writing the contract each period. (This cost is incurred each period.) The players discount the future with a common discount factor, denoted δ.

If the contract is breached, the players go to court. This is costless. With probability v, where $0 \leq v \leq 1$, the court observes both the level of q that is produced and whether t has been paid. If the court observes this, it will enforce the contract. With probability $1 - v$ the court is unable to make the relevant observation, and then the case is dismissed. If the court enforces the contract, it requires the breaching player to pay the other an amount that gives the non-breaching player what she expected to receive under the contract. (If the agent does not produce the good of the agreed upon quality q, she must pay an amount that gives the principal $q - t$. If the principal breaches, say by not paying or only partially paying, she must pay an amount that gives t to the agent.)

Suppose that the players agree on the same value of q and t for each period. Under what conditions does there exist a subgame perfect equilibrium in the infinitely-repeated game? (Use trigger strategies.) Explain.

Problem 5 (40 %)

Consider the standard moral hazard model: An agent, who is risk neutral or risk averse, shall do a job for a risk-neutral and profit-maximizing principal. The agent can exert high or low effort; the outcome can be good or bad for the principal. The effort is not observable, but the outcome is. The probability of a good outcome is greater for high effort than for low, but high effort is more costly for the agent.

When answering the questions below, you may use the following notation:

- The gross (monetary) value to the principal of a good outcome is $\overline{S}$, while the value of a bad outcome is $\underline{S}$, where $\overline{S} > \underline{S}$. From this gross value is subtracted any transfer paid to the agent, denoted t.
- High effort is denoted $e = 1$ while low effort is denoted $e = 0$.
- The agent's utility has the form $u(t) - \psi(e)$, where t is the transfer and e is the effort. The function u satisfies $u' > 0$ and $u'' \leq 0$. The function ψ can be normalized by setting $\psi(0) = 0$ and $\psi(1) = \psi > 0$.
- The probability of a good outcome given effort level e is denoted π_e, with $1 > \pi_1 > \pi_0 > 0$.

(a) What is, in this connection, meant by the term "participation constraint"?

(b) Explain why it is socially optimal that the agent exert high effort if and only if

$$\left(\overline{S} - \underline{S}\right)\left(\pi_1 - \pi_0\right) \geq u^{-1}(\psi) - u^{-1}(0)$$

(c) If the condition of (b) is not satisfied, what kind of a contract should the principal offer to the agent?

(d) Then assume that the condition of (b) is satisfied, and the agent is risk neutral. Describe the contract offered by the principal in such a case. Discuss the realism of implementing such a contract.

(e) Again assume that the condition of (b) is satisfied, but the agent is now risk averse. Describe the conditions a contract must satisfy in order to induce high effort. Would the principal always want to offer such a contract?

(f) Under the conditions of (e), the socially optimal ("first-best") outcome cannot be achieved. Explain why.